# Fast deterministic numerical methods for the quantum Boltzmann-Nordheim equation

Alexandre Mouton[1,2] and Thomas Rey[1,2]

[1] *Laboratoire Paul Painlevé, CNRS & Université de Lille, France*
[2] *Inria Lille Nord Europe, France*

**Keywords:** Rarefied gases kinetic modeling, Bose-Einstein condensation, Fermi-Dirac saturation, Fourier-Galerkin method, high performance computing.

Spectral methods, thanks to their high accuracy and the possibility to use fast algorithms, represent an effective way to approximate the collisional kinetic equations of Boltzmann type, such as the Boltzmann-Nordheim equation (BNE). This equation, modeled on the seminal Boltzmann equation, describes using a statistical physics formalism the time evolution of a gas composed of bosons or fermions and is written in $d_{\mathbf{x}} + d_{\mathbf{v}} + 1$ dimensions in the time-phase space with $d_{\mathbf{x}} = 1, 2, 3$, $d_{\mathbf{v}} = 2, 3$. Using the Fourier-Galerkin algorithm introduced in [1], we investigate some of the conjectured properties of the large time behavior of the solutions to the spatially homogeneous Boltzmann-Nordheim equation (SHBNE) that is written in 2D or 3D in velocity + time (see [3]).
Due to the trilinear operators embedded in the collision operator of (SHBNE), the computational complexity of such numerical scheme is of the same order as $\mathcal{O}(N^{2d_{\mathbf{v}}} \log(N))$ when a $N^{d_{\mathbf{v}}}$ grid is considered. Hence parallelizing these computations is required to produce accurate results within a reasonable time cost and if we want to adapt our code to the inhomogeneous equation (BNE).
Finally, we present some recent improvements of our code that are inspired by [2]: this allows to reduce the computational complexity to the order $\mathcal{O}(MN^{d_{\mathbf{v}}} \log(N))$ where $M$ stands for the number of Galerkin integration points used for computing the cross-modes. Up to a good choice of $M$, we show that this scalability improvement may not come with a loss of accuracy.

This work is partially funded by Labex CEMPI (ANR-11-LABX-0007-01) and ANR Project MoHyCon (ANR-17-CE40-0027-01). It is also been funded by PRACE Summer of HPC 2021 programme (internships of D. Knapp and A. Mavliutov).

## References

[1] Francis Filbet, Jingwei Hu, and Shi Jin. "A Numerical Scheme for the Quantum Boltzmann Equation Efficient in the Fluid Regim". In: *ESAIM Math. Model. Numer. Anal.* 42 (2012), pp. 443–463.

[2] Irene Gamba et al. "A Fast Spectral Method for the Boltzmann Collision Operator with General Collision Kernels". In: *SIAM J. Sci. Comput.* (2017), B658–B674.

[3] Alexandre Mouton and Thomas Rey. *On Deterministic Numerical Methods for the quantum Boltzmann-Nordheim Equation - I. Spectrally accurate approximations, Bose-Einstein condensation, Fermi-Dirac saturation.* (submitted). arXiv: 2110.13735.