# KINEBEC - Numerical simulation of Boltzmann-Nordheim equation

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## About the project

KINEBEC project started on November 2019 and is dedicated to the numerical study of Boltzmann-Nordheim equation (BNE). This kinetic model has been derived by Uehling and Uhlenbeck in 1933 and is designed for describing a gas of quantum particles (bosons or fermions) in which the temperature is assumed to be very small such that collisions between particles embed quantum effects. This physical phenomenon can be represented by an additional trilinear operator in the collision term and requires important numerical ressources to be accurately discretized. In addition, this modified collision term may induce degenerative time behaviour of the distribution function of the gas, such as Bose-Einstein condensation or Fermi-Dirac saturation. For this purpose, we aim to develop a simulation code that is highly accurate, scalable and easy to use.

## A brief timeline

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| Nov. 2019 | Start of the project with A. Mouton and T. Rey |
| Mar. 2020 | Beginning of implementation of the code for solving the spatially homogeneous BNE<br>MPI parallelization of the collisions |
| Jul. 2020 | Implementation of velocity rescaling method |
| Sept. 2020 | OpenMP parallelization of the collision computations<br>Definition of protocols for numerical validation of the code based on the classification of time relaxation behaviors |
| Apr. 2021 | Cuda parallelization of the collisions computations |
| Summer 2021 | Internships of D. Knapp and A. Mavliutov (PRACE SoHPC fundings)<br>• Testing new MPI parallelization methods<br>• First tests on the inhomogeneous BNE (2D in space + 2D in velocity) |
| Oct. 2021 | Implementation of fast Fourier-Galerkin methods |

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## Fundings

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A financial partner with PRACE has also been established for funding the internships of D. Knapp and A. Mavliutov in the context of the programme Summer of HPC 2021.

## Publications

[1] Alexandre Mouton and Thomas Rey. *On Deterministic Numerical Methods for the quantum Boltzmann-Nordheim Equation - I. Spectrally accurate approximations, Bose-Einstein condensation, Fermi-Dirac saturation.* (submitted). arXiv: 2110.13735.

[2] Alexandre Mouton and Thomas Rey. *On Deterministic Numerical Methods for the quantum Boltzmann-Nordheim Equation - II. Fast algorithms.* work in progress.

## Communications

1. Mouton, A.: *On Deterministic Numerical Methods for the quantum Boltzmann-Nordheim Equation*
   Workshop "Asymptotic behaviour of systems of PDEs arising in physics and biology: theoretical and numerical point of view - 4th ed.", Lille (France), 16/11/2021 - 19/11/2021
2. Rey, T.: *On deterministic numerical methods for solving collision kinetic equations of Boltzmann type*
   Annual British Applied Mathematics Colloquium, Cambridge (UK), 8/4/2021.
3. Rey, T.: *Des méthodes spectrales pour résoudre numériquement l'équation de Boltzmann*
   Journée d'Analyse Appliquée des Hauts de France, Lille (France), 26/11/2020
4. Rey, T.: *On deterministic numerical methods for solving collision kinetic equations of Boltzmann type*
   Seminar of the Cambridge Kinetic Group, Cambridge (UK), 14/10/2020.

## Websites

Webpage of KINEBEC project:

- https://sites.google.com/site/moutonalexandre/research/kinebec

Link to the code:

- https://hal.archives-ouvertes.fr/hal-03464411

Webpage of SoHPC 2021:

- https://summerofhpc.prace-ri.eu

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