CSC/PRACE Spring School in Computational Chemistry 2020

Introduction to Electronic Structure Theory

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Part II: Density Functional Theory

The basic ideas of DFT

- The foundation for contemporary DFT is the Hohenberg–Kohn theorem (1964)
	- o *The energy of a molecular system, as well as all other observables are unambiguously defined by the electron density of the system*
- · Implication: No direct knowledge of the wave function is necessary, and thus, **no need to solve the Schrödinger equation**
- · An exact solution of the SE requires, in principle, a computational effort scaling **exponentially** with the number of electrons
	- \circ The dimensionality of FCI is approximately $[N!/(n/2)! \cdot (N-n/2)!]^2$ *N* = number of orbitals,

n = number of electrons

· In contrast, the equations of the perfect density functionals should require an effort **independent of the number of electrons**; the dimensionality would be 3.

o The development of functionals are nowhere near this nirvana

- · Next, we will have a quick look at different density functional types in use today
	- o pre-HK DFT (Thomas–Fermi, Dirac) will be left for self-study

The potential for the ground state of a finite system is directly (up to a constant) defined by the electron density

Proof: let v(r) be the potential and ρ(r) the electron density. If the HK theorem would *not* be true, another potential v'(r), where **v'(r) ≠ v(r) + constant**, giving the same ρ(r) should exist. Thus, also two different wave functions, Ψ and Ψ', corresponding to the external potential v and v' would exist

The variational principle: $E_0 = \langle \Psi | H | \Psi \rangle$ < $\langle \Phi | H | \Phi \rangle$, Ψ is the exact wf, Φ not

Now, with ρ(r) and ρ'(r) identical, identical kinetic energies and electron-electron interaction for H and H' \rightarrow **E⁰ = <Ψ|H|Ψ> <** <Ψ'|H|Ψ'> = <Ψ'|H-H'+H'|Ψ'> = **<Ψ'|H'|Ψ'>** + <Ψ'|H-H'|Ψ'> = **E'0** + <Ψ'|H-H'|Ψ'> *but also*:

E'₀ = $\langle \Psi' | H' | \Psi' \rangle$ < $\langle \Psi | H' | \Psi \rangle$ = $\langle \Psi | H | \Psi \rangle$ + $\langle \Psi | H' - H | \Psi \rangle$ = $E_0 + \langle \Psi | H' - H | \Psi \rangle$

· The above *cannot be true*, as it implies **E0 > E'0 > E⁰**

The Hohenberg–Kohn theorem according to E.B. Wilson

Another way of looking at it:

- **1)** The electron density ρ(r) contains **the number of electrons** in the system
- **2)** Cusps in ρ(r) appear at atomic nuclei, defining **the position of atoms**
- **3)** The forms of the cusps define the number of protons, that is, **the atom types**

We note that in order to define the molecular electronic Hamiltonian, only the number of electrons and the atomic coordinates, which make up the external potential, are needed; **we have everything in ρ(r)!**

$$
Z_k=-\frac{a_o}{2n({\bf r})}\frac{dn({\bf r})}{dr}|_{r\rightarrow {\bf R_k}}
$$

· Every specific electron density gives a specific energy (for the GS), the energy is a **functional of ρ**

```
electronic kinetic energy
electron-electron repulsion, J[ρ]-K[ρ]
                  \sqrt{1 + \frac{1}{2}}E[ρ] = T[ρ] - E<sub>ne</sub>[ρ] + E<sub>ee</sub>[ρ]
```
electron-nucleus attraction

- · The main problem of early density functionals was a **poor description of the kinetic energy** when modelled by the total density alone
	- o Difficult to "see" the movement of individual electrons from the total density alone!
- · Kohn and Sham presented a formalism, **based on orbitals**, for treating the kinetic energy

· Idea based on **Hartree's model** where the electrons move in an effective potential created by the nuclei and the *mean field* created by the other electrons

$$
v_{\rm H}(\mathbf{r})=-\sum_a \frac{Z_a}{|\mathbf{R}_a-\mathbf{r}|}+\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}{\rm d}\mathbf{r}' \qquad \left[-\frac{1}{2}\nabla^2+v_{\rm H}(\mathbf{r})\right]\psi_i=e_i\psi_i
$$

- · In **Kohn–Sham DFT**, a system of **independent non-interacting electrons** in a common one-body potential, v_{KS} , is imagined
- · KS also introduced orbitals into DFT, originally *assumed* to be independent reference orbitals fulfilling the Schrödinger equation for independent particles:

$$
\left[-\frac{1}{2}\nabla^2 + v_{\rm KS}\right]\psi_i = e_i\psi_i
$$

- · The introduction of orbitals **increases the dimensionality of DFT** from 3 to **3***N*
- · This is more than compensated for by a **much-improved** description of the **kinetic energy**
	- o Still, dimensionality the same as for the simplest wave function methods!
- · The KE for the **non-interacting** electrons is then (lower index s denotes *single-*electron equations):

$$
T_{\rm s}[\rho]=\sum_{i=1}^N\langle\psi_i|-\frac{1}{2}\nabla^2|\psi_i\rangle
$$

· Electrons of course *do* interact, and the missing part is denoted the correlation kinetic energy

$$
T_{\rm c}[\rho]=T[\rho]-T_{\rm s}[\rho]\geq 0
$$

- · *T*c is usually included in an **exchange/correlation** term *E*xc
	- \circ The amount of kinetic correlation energy is of the same order of magnitude as the total correlation energy, but always of opposite sign
- · Now, the KS equations can be solved analogously to the SCF Hartree equations

· Within KS-DFT, the energy of the ground state, divided into its components, is then given by:

$$
E^{\rm DFT}[\rho] = T_{\rm s}[\rho] + E_{\rm ne}[\rho] + J[\rho] + E_{\rm xc}[\rho]
$$

- · We now have an **exact energy expression**
- · Further, of the terms, all but the last, the exchange/correlation energy, *can* be solved exactly
- · Kohn and Sham paved the way for a renaissance for DFT
	- o The problem of the kinetic energy was largely solved
- · **New challenge**: Find a solution for *E*xc

Different DFT models

- · In wave function theory, there is a **systematic way of improving the quality of the model**
	- o Not much joy if the systems are so large that nothing proper can be performed...
- · Within DFT, **the exact functional really** *is* **unknown**
	- o Some **constraints** on properties the functional should fulfil *are* known
- · Hierarchies of complexity *do* exist also within **DFT**
- · The idea is to include **more complex properties** of the electron density into the description
- · Climbing **Jacob's ladder of DFT**: each rung bringing the functional closer to perfection
	- o Perdew *et al,"* Some Fundamental Issues in Ground-State Density Functional Theory: A Guide for the Perplexed", *J. Chem. Theory Comput.* **5** (2009) 902, *http://dx.doi.org/10.1021/ct800531s*
	- o Mardirossian & Head-Gordon (2017) "Thirty years of density functional theory in computational chemistry: an overview and extensive assessment of 200 density functionals"

https://doi.org/10.1080/00268976.2017.1333644

The Local Density Approximation (LDA)

- Takes only the electron density in specific points in space into account $\epsilon^{LSDA} = \epsilon^{LSDA}(\rho_\alpha, \rho_\beta)$
- · In LDA, the electron density is assumed to vary slowly in space

$$
E_{\rm xc}^{\rm LDA}[\rho]=\int \rho({\bf r})\varepsilon_{\rm xc}^{\rm unif}(\rho)
$$

• The exchange energy of a uniform electron gas is analytically known (Slater/Dirac/Bloch exchange)

$$
E_{\rm X}^{\rm LDA}[\rho] = -C_{\rm X} \int \rho^{4/3}(\mathbf{r}) d\mathbf{r}, \quad C_{\rm X} = \frac{3}{4} \left(\frac{3}{\pi}\right)^{1/3}
$$

- This is where the train stops for analytically derived DFT
- There is no known equation for the correlation energy for even such a simple model system as the uniform electron gas!
	- o It can, however, be computed very accurately using quantum Monte Carlo, and numerical fits to the results can be formulated
- · The fact remains that already the LDA correlation functionals are nothing but *ad hoc* functionals with no real physical meaning except that they provide good results

Chemically useful approximations

- · **LDA is not accurate enough for chemistry**
	- o On rare occasions, it *seems* to be, but only due to heavy error cancellation
- · In order to construct more accurate functionals, one notes that **ρ(r) contains** *much* **more information than just the electron density at specific points**
- Increased accuracy (usually) comes at a price: Climbing the ladder makes the calculations more expensive!

The Generalised Gradient Approximation

- · **The electron density is** *not* **uniform**
- GGAs account for this by also considering the gradient of the density $\nabla \rho$ into account
	- o Introduced in 1986 by Perdew and Wang; before, gradients had only been considered to second order, $|\nabla \rho|^2$
	- o Term generalised comes from the GGAs considering higher powers of $|\nabla \rho|$ into account; *generally*, any power
- A general GGA thus has the form $\epsilon^{GGA} = \epsilon^{GGA}(\rho_\alpha, \rho_\beta, \nabla \rho_\alpha, \nabla \rho_\beta)$
- · GGAs are **semi-local**

• Usually build upon the LDA expressions:
$$
E_{\rm x}^{\rm B88} = E_{\rm x}^{\rm LSDA} - \beta \sum_{\sigma} \int \rho_{\sigma}^{4/3} \frac{x_{\sigma}^2}{1 + 6\beta x_{\sigma} \sinh^{-1} x_{\sigma}} dr
$$

$$
\varepsilon_{\rm c}^{\rm PBE} (r_{\rm s}, \zeta, t) = \varepsilon_{\rm c}^{\rm PWB2} (r_{\rm s}, \zeta) + H (r_{\rm s}, \zeta, t)
$$

 Ω

Meta-GGAs

· In addition to ρ and Ñρ, also the **Laplacian** Ñ**²ρ** and/or **the kinetic energy density τ** considered

$$
\varepsilon^{\text{mGGA}} = \varepsilon^{\text{mGGA}}(\rho_\alpha,\rho_\beta, \nabla \rho_\alpha, \nabla \rho_\beta, \nabla^2 \rho_\alpha, \nabla^2 \rho_\beta, \tau_\alpha, \tau_\beta) \quad \tau_\sigma(\mathbf{r}) = \frac{1}{2} \sum_i^{ \text{occ} } |\nabla \psi_{i\sigma}(\mathbf{r})|^2
$$

· τ depends on the KS orbitals, meta-GGAs that use τ are thus **non-local**

Hybrid functionals

· Hartree–Fock can in principle provide the **exact exchange** energy via the orbitals

$$
E_\mathrm{x}^\mathrm{HF} = -\frac{1}{2}\sum_i^n\sum_j^n\int\int\frac{\psi_i(\mathbf{r}_1)\psi_j(\mathbf{r}_1)\psi_i(\mathbf{r}_2)\psi_j(\mathbf{r}_2)}{|\mathbf{r}_1-\mathbf{r}_2|}\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2
$$

- · A **hybrid method** combines HF-like exchange energy with a DFT description of *E*xc
- The simplest hybrid would just take HF exchange and DFT correlation $\ E_{\rm xc}=E_{\rm v}^{\rm HF}+E_{\rm c}^{\rm DFT}$ o Too simple, doesn't work well
- · Combining **a fraction** of HF-like exchange improves thermochemical results **dramatically**
- · **B3LYP** (1993!) made many chemists true believers in the power of DFT

$$
E_{\rm xc}^{\rm B3LYP} = a_0 E_{\rm x}^{\rm HF} + (1 - a_0) E_{\rm x}^{\rm LSDA} + a_{\rm x} \Delta E_{\rm x}^{\rm B88} + (1 - a_{\rm c}) E_{\rm c}^{\rm VWN} + a_{\rm c} E_{\rm c}^{\rm LYP}
$$

- · Others found the fraction Frankensteinian:
	- o P. Gill, "Obituary: Density Functional Theory (1927-1993)", *Aust. J. Chem.* **54** (2001) 661, *http://dx.doi.org/10.1071/CH02049*

Functional development philosophies

- · Even with the **ingredients** of different levels of DFT in place, the actual **recipe** on how to use them is completely open
- · Different approaches exist
	- o Invent a functional form that reproduces wanted data: **empirical**
	- o Invent a functional form that fulfils known properties of the true functional: **non-empirical**
	- o Use both approaches; often starting from a non-empirical formulation and slightly adjusting it for pragmatic reasons
- · Empirical functionals usually work well **for systems similar to those parameterised for**
	- o Can fail spectacularly when outside their comfort region
- · Non-empirical functionals usually perform less well
	- o But without parameters for specific systems, can be hoped to perform equally well for "everything"

Non-empirical functionals

- · **LDA is usually non-empirical**
- · GGAs and meta-GGAs come in many forms, most of which have at least some parameters fitted to experimental data
- · The typical non-empirical GGA is **PBE**, for solids **PBEsol** is better
- · The typical non-empirical meta-GGA is **TPSS**, an even better one is **revTPSS:**
	- o Perdew *et al,* "Workhorse Semilocal Density Functional for Condensed Matter Physics and Quantum Chemistry", *PRL* **103** 026403, *http://dx.doi.org/10.1103/PhysRevLett.103.026403*
- · **A closer look at PBE**, *Phys. Rev. Lett.* **77** (1996) 3865, *http://dx.doi.org/10.1103/PhysRevLett.77.3865*

Generalized Gradient Approximation Made Simple

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Generalized gradient approximations (GGA's) for the exchange-correlation energy improve upon the local spin density (LSD) description of atoms, molecules, and solids. We present a simple derivation of a simple GGA, in which all parameters (other than those in LSD) are fundamental constants. Only general features of the detailed construction underlying the Perdew-Wang 1991 (PW91) GGA are invoked. Improvements over PW91 include an accurate description of the linear response of the uniform electron gas, correct behavior under uniform scaling, and a smoother potential. [S0031-9007(96)01479-2]

- · One motivation for the construction was to **simplify the non-empirical PW91 functional**
- · For PBE, only conditions that were considered energetically important are satisfied
	- o Less important conditions are ignored

Next up, a **quick non-detailed overview of the "derivation**"

$$
\varepsilon_{\mathrm{c}}^{\mathrm{PBE}}\left(r_{\mathrm{s}},\zeta,t\right)=\varepsilon_{\mathrm{c}}^{\mathrm{PW92}}\left(r_{\mathrm{s}},\zeta\right)+H\left(r_{\mathrm{s}},\zeta,t\right)
$$

 $\mathcal{H} = \left| \nabla \rho \right| / \left(2g k_{\rm s} \rho \right)$ dimensionless density gradient

 $r_{\rm s}=\left(3/\left(4\pi\rho\right)\right)^{1/3}$ Wigner-Seitz radius (avg. radius containing one electron)

 $\zeta=\left(\rho_{\uparrow}-\rho_{\downarrow}\right)/\rho$ relative spin polarisation

· Builds upon LDA (specified as PW92 LDA)

- · **Three exact conditions are satisfied**
- **1.** In the slowly varying limit (*t* --> 0), *H* should go to

$$
H \to (e^2/a_0) \beta \phi^3 t^2
$$

2. In the rapidly varying limit $(t \rightarrow \infty)$

$$
H \to -\epsilon_{\rm C}^{\rm unif}
$$

This makes correlation vanish

3. Under uniform scaling, the correlation energy must scale to a constant

$$
\rho_{\lambda}(x, y, z) = \lambda^{3} \rho(\lambda x, \lambda y, \lambda z)
$$

\n
$$
\lim_{\lambda \to \infty} E_{c}[\rho_{\lambda}] = \text{const} > -\infty
$$

To achieve this, *H* must cancel the logarithmic singularity of ε_c ^{LDA}

· All the above three conditions are satisfied by the following form for *H:*

$$
H = (e2/a0)\gamma\phi3
$$

$$
\times \ln\left\{1 + \frac{\beta}{\gamma}t^{2}\left[\frac{1 + At^{2}}{1 + At^{2} + A^{2}t^{4}}\right]\right\},
$$

where

$$
A = \frac{\beta}{\gamma} \left[\exp\{-\epsilon_{\rm C}^{\rm unif}/(\gamma \phi^3 e^2/a_0) \right\} - 1 \right]^{-1}.
$$

- When *t*=0, *H* is exactly condition 1, when *t*-->_∞, *H* grows monotonically to the limit of condition 2
- \bullet Thus, $E_{\text{C}}^{\text{GGA}} \leq 0$

· **Compared to PW91**, quite much simpler:

$$
\varepsilon_{\mathrm{c}}^{\mathrm{PW91}}\left(r_{\mathrm{s}},\zeta,t\right)=\varepsilon_{\mathrm{c}}^{\mathrm{PW92}}\left(r_{\mathrm{s}},\zeta\right)+H_{0}\left(r_{\mathrm{s}},\zeta,t\right)+H_{1}\left(r_{\mathrm{s}},\zeta,t\right)
$$

$$
H_0(r_s, \zeta, t) = \frac{g^3 \beta^2}{2\alpha} \ln\left(1 + \frac{2\alpha}{\beta} \frac{t^2 + At^4}{1 + At^2 + A^2 t^4}\right)
$$

$$
A = \frac{2\alpha}{\beta} \frac{1}{e^{-2\alpha \varepsilon_c^{\text{PW92}}/(g^3 \beta^2)} - 1}
$$

$$
H_{1}\left(r_{\rm s}, \zeta, t\right) = \nu \left(C_{\rm c}\left(r_{\rm s}\right) - C_{\rm c}(0) - \frac{3}{7}c_{\rm x}\right)g^{3}t^{2}e^{-100g^{4}\left(k_{\rm s}^{2}/k_{\rm F}^{2}\right)t^{2}}
$$
\n
$$
C_{\rm c}\left(r_{\rm s}\right) = \frac{0.002568 + 0.023266r_{\rm s} + 7.389 \cdot 10^{-6}r_{\rm s}^{2}}{1 + 8.723r_{\rm s} + 0.472r_{\rm s}^{2} + 0.07389r_{\rm s}^{3}} + 0.001667
$$

PBE exchange

$$
E_{\rm x}^{\rm GGA}[\rho]=\int f\left(\rho({\bf r}),\left|\nabla\rho({\bf r})\right|\right)d^3r=-C_{\rm x}\int\rho^{4/3}({\bf r})F(s({\bf r}))d^3r
$$

$$
s=\left|\nabla\rho\right|/\left(2\rho k_{\rm F}\right)~{\rm with}~k_{\rm F}=\left(3\pi^2\rho\right)^{1/3}
$$

· **Based on four additional conditions**

Satisfied by the simple

$$
F^{\rm PBE}(s) = 1 + 0.804 - \frac{0.804}{1 + \frac{0.21951}{0.804}s^2}
$$

Again, quite much simpler than PW91:

$$
F^{\mathrm{PW91}}(s) = \frac{1 + 0.19645s \: \mathrm{arcsinh}(7.7956s) + \left(0.2743 - 0.1508e^{-100s^2}\right)s^2}{1 + 0.19645s \: \mathrm{arcsinh}(7.7956s) + 0.004s^4}
$$

DFT for dispersion

- · **What is dispersion interaction?**
- · **Attraction** between neutral fragments due to polarisation caused by quantum fluctuations

- · Also known as **van der Waals** and **London** forces
- Decays as R^{-6} , strength depends on the IP and polarisability of the fragments (London, 1930):

- · *Nonlocal* phenomenon, no overlap of electron densities needed
- · The functionals we have seen so far are (semi)local, at least up to GGA level
	- \circ *E*[LDA] = *E*[ρ]
	- o $E[\text{GGA}] = E[\rho, |\nabla \rho|^{n}]$
	- o *E*[m-GGA] = *E*[ρ, |Ñρ| n , |^Ñ ²ρ|, τ] (**τ orbital dependent, though!**)
	- o hybrids don't help, **vdW is correlation**
- · Therefore, **there is no reason, even possibility** for vdW forces to be described well by semi-local functionals

DFT for dispersion

Some attempts to modify (reparametrize) existing functionals

• Cannot really work if the necessary physical information is missing!

FIG. 3. Two binding configurations of the DNA base pairs adenine and thymine. A hydrogen bonded structure is shown on the left (hydrogen bonds indicated by red dots) and a "stacked" geometry on the right. For the hy-

DFT for dispersion – double hybrids

Incorporation of **correlated WF methods (MP2)** has also been used

THE JOURNAL OF CHEMICAL PHYSICS 124, 034108 (2006)

Semiempirical hybrid density functional with perturbative second-order correlation

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The uniformity with

which B2-PLYP improves for a wide range of chemical systems emphasizes the need of (virtual) orbital-dependent terms that describe nonlocal electron correlation in accurate exchange-correlation functionals. From a practical point of view, the new functional seems to be very robust and it is thus suggested as an efficient quantum chemical method of general purpose. © 2006 American Institute of Physics. [DOI: 10.1063/1.2148954]

- · **B2-PLYP**, Grimme *J. Chem. Phys.* **124** (2006) 034108
	- o Based on the B88 exchange functional and the LYP correlation functional (BLYP)
- HF exact exchange added
- · Second order perturbation (PT2/MP2) added: It is thus a **double-hybrid functional**

DFT for dispersion – B2PLYP

· On the **fifth rung of Jacob's ladder**, as it takes virtual orbitals into account

Our much simpler ansatz for a combination of KS-DFT and PT is based on the following expression for the exchange-correlation energy E_{xc} and is given by $E_{\text{xc}} = (1 - a_x)E_x^{\text{GGA}} + a_x E_x^{\text{HF}} + bE_c^{\text{GGA}} + cE_c^{\text{PT2}},$ (1) where (in spin-orbital form) $E_c^{\text{PT2}} = \frac{1}{4} \sum_{ia} \sum_{ib} \frac{[(ia|jb) - (ib|ja)]^2}{\epsilon_i + \epsilon_i - \epsilon_a - \epsilon_b}.$ (2)

DFT for dispersion – B2PLYP

- · "**Drawbacks" of B2-PLYP compared to "normal" DFT**
- · Higher basis set demand
	- \circ The virtual space in the PT2 treatment requires larger basis sets, just as normal WF MP2
	- o Minimum recommended: TZVPP
	- o **"**I would consider an B2PLYP/6-31g* type calculation as almost useless", Grimme, CCL 16 Oct 2009
- · Somewhat larger computational cost
	- \circ Compared to other hybrids, not that bad, as the MP2 term can be computed quite efficiently with RI (RI-B2-PLYP)
- · **Still not that good for long-range dispersion!**
	- o PT2 part relatively small compared to the poorly performing LYP correlation
- · Overall, **seems to work quite well**, however
- · Newer double hybrids among the best for thermochemistry *for molecules made from main-group elements*

Empirical force-field type dispersion on top of DFT: DFT-D

- · MM force fields can perform much better for dispersion than DFT, at least for dispersion
- The R^{-6} term is simply one of the force field parameters
- · As dispersion is long-range, it usually has a very **small effect on the total density**
- · This motivates the general form of DFT-D

$$
E_{\text{DFT}-\text{D}} = E_{\text{KS}-\text{DFT}} + E_{\text{disp}}
$$

- · The dispersion correction is just **added on top** of the normal DFT calculation
- · The potential energy surface is thus modified, and better geometries and binding energies should then be obtained

DFT-D

· The form of *E*disp is relatively simple (Grimme, *J. Comput. Chem.* 27 (2006) 1787 (actually, the second incarnation, DFT-D2):

$$
E_{\rm disp} = -s_6 \sum_{i=1}^{N_{\rm at}-1} \sum_{j=i+1}^{N_{\rm at}} \frac{C_6^{ij}}{R_{ij}^6} f_{\rm dmp}(R_{ij})
$$

*N***at** is the number of atoms C_6 are atomic dispersion coefficients, and $C_6^{ij} = \sqrt{C_6^i C_6^j}$

s6 is a functional dependent **global scaling factor**

DFT-D

$$
E_{\text{disp}} = -s_6 \sum_{i=1}^{N_{\text{at}}-1} \sum_{j=i+1}^{N_{\text{at}}} \frac{C_6^{ij}}{R_{ij}^6} f_{\text{dmp}}(R_{ij})
$$

The **damping function** f_{dmp} is compulsory to avoid near-singularities for small *R*

- · This would lead to infinite attraction...
- · It also ensures that vdW correction takes place at the distances which are relevant **and neglected by normal DFT**, that is, long-range interaction where e-density overlap is small

At short distances, the *R* –6 behaviour is not valid anymore, either

Figure 1. Dispersion contributions to the interaction potentials for two carbon atoms ($s_6 = 1.0$) separated by a distance R. The dashed line shows the undamped potential for comparison.

DFT-D

• The damping function has the form:
$$
f_{\text{dmp}}(R_{ij}) = \frac{1}{1 + e^{-d(R_{ij}/R_r - 1)}}
$$

- · The problem of **double-counting correlation** is still real, even after damping!
	- o "Fixed" by the scaling parameter *s***⁶**
	- \circ s₆ is fitted to 40 non-covalently bound complexes
		- § PBE: 0.75
		- § BLYP: 1.2
		- § BP86: 1.05
		- TPSS: 1.0
		- § B3LYP: 1.05
		- § B97-D: 1.25
		- B2PLYP: 0.55 ← dispersion already in via PT2 (note: *triple-counting* of correlation...)

Performance of DFT-D

· **DFT-D usually works quite well!**

FIGURE 3. Histogram of deviations with respect to experimental data for the G3/99 set of heats of formation. The corresponding MAD values are 5.6 kcal mol⁻¹ (B3LYP), 3.1 kcal mol⁻¹ (B3LYP-D), 2.4 kcal mol⁻¹ (B2PLYP), and 1.7 kcal mol⁻¹ (B2PLYP-D).

Average signed errors for H-bonded, dispersion bonded, and "mixed" interaction energies from the S22 set; kcal/mol, DFT / **DFT-D** (*J. Comput. Chem.* **28** (2007) 555)

BUT: DFT-D is not the final solution!

- Just as with force fields, it works well for the types of systems it was designed for
- The possible double counting of correlation is ever present
- · **There is no way to know exactly what is missing in DFT**, and thus adding "something" on top can (*will*) fail

DFT-D3

- Latest *commonly available* version (2010) of Grimme's scheme with less empiricism and more geometry dependence
- · **Recommended.** *http://dx.doi.org/10.1063/1.3382344*

THE JOURNAL OF CHEMICAL PHYSICS 132, 154104 (2010)

A consistent and accurate ab initio parametrization of density functional dispersion correction (DFT-D) for the 94 elements H-Pu

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(Received 18 January 2010; accepted 16 March 2010; published online 16 April 2010)

The method of dispersion correction as an add-on to standard Kohn–Sham density functional theory (DFT-D) has been refined regarding higher accuracy, broader range of applicability, and less empiricism. The main new ingredients are atom-pairwise specific dispersion coefficients and cutoff radii that are both computed from first principles. The coefficients for new eighth-order dispersion terms are computed using established recursion relations. System (geometry) dependent information is used for the first time in a DFT-D type approach by employing the new concept of fractional coordination numbers (CN). They are used to interpolate between dispersion coefficients of atoms

FIG. 7. Left: Comparison of MAD values for different functionals without dispersion correction (DFT), with the old (DFT-D2), and new (DFT-D3) versions. Right: MAD values averaged over nine DFs (excluding BP86 and PBE) for the different subsets.

DFT-D4

- Latest version (2019) of Grimme's scheme
- · Adds dependence on atomic charge for the dipole polarizabilities in the DFT-D3 model
- Will find its way to the major QC packages soon enough

A Generally Applicable Atomic-Charge Dependent **London Dispersion Correction Scheme**

Version 2 \vee Preprint revised on 25.01.2019, 17:50 and posted on 25.01.2019, 17:57 by Eike Caldeweyher, Sebastian Ehlert, Andreas Hansen, Hagen Neugebauer, Sebastian Spicher, Christoph Bannwarth, Stefan Grimme

The D4 model is presented for the accurate computation of London dispersion interactions in density functional theory approximations (DFT-D4) and generally for atomistic modeling methods. In this successor to the DFT-D3 model, the atomic coordination-dependent dipole polarizabilities are scaled based on atomic partial charges which can be taken from various sources. For this purpose, a new chargedependent parameter-economic scaling function is designed. Classical charges are obtained from an atomic electronegativity equilibration procedure for which efficient

The Random Phase Approximation (RPA)

- · The RPA idea is old, from the 1950's
- Used to be much too expensive
- · In 2008, Furche reformulated the RPA into a useably efficient form

DOI 10.1007/s00214-011-1084-8 **REGULAR ARTICLE**

Theor Chem Acc (2012) 131:1084

Electron correlation methods based on the random phase approximation

Henk Eshuis · Jefferson E. Bates · Filipp Furche

- o "Developing the random phase approximation into a practical post-Kohn–Sham correlation model", *J. Chem. Phys.* **129** (2008) 114105, *http://dx.doi.org/10.1063/1.2977789*
- · **Non-empirical**

Fig. 5 RPA mean errors (ME), mean absolute errors (MAE), and 4 maximum absolute errors (Max) (kcal/mol) in the relative energies of n -alkane conformers compared to other methods. Geometries and

Fig. 6 RPA mean absolute percentage errors (MA%E), mean errors (ME), mean absolute errors (MAE), and maximum absolute errors (Max) (kcal/mol) for the S22 test set compared to other methods.

Published: 11 October 2017 Article **OPEN**

Bypassing the Kohn-Sham equations with machine learning

Felix Brockherde, Leslie Vogt, Li Li, Mark E. Tuckerman M, Kieron Burke & Klaus-Robert Müller

Nature Communications 8, Article number: 872 (2017) | Download Citation \pm

Abstract

Last year, at least 30,000 scientific papers used the Kohn-Sham scheme of density functional theory to solve electronic structure problems in a wide variety of scientific fields. Machine learning holds the promise of learning the energy functional via examples, bypassing the need to solve the Kohn-Sham equations. This should yield substantial savings in computer time, allowing larger systems and/or longer time-scales to be tackled, but attempts to machine-learn this functional have been limited by the need to find its derivative. The present work overcomes

Semi-local machine-learned kinetic energy density functional with thirdorder gradients of electron density

J. Chem. Phys. 148, 241705 (2018); https://doi.org/10.1063/1.5007230

Correction: J. Chem. Phys. 149, 079901 (2018)

ABSTRACT

A semi-local kinetic energy density functional (KEDF) was constructed based on machine learning (ML). The present scheme adopts electron densities and their gradients up to third-order as the explanatory variables for ML and the Kohn-Sham (KS) kinetic energy density as the response variable in atoms and

What density functional to choose?

- · **Search the literature for benchmark studies** of similar systems!
- · If you are looking at something new, you **really need to do** *some* **benchmarking** yourself

Cite this: Phys. Chem. Chem. Phys., 2017.19.32184

A look at the density functional theory zoo with the advanced GMTKN55 database for general main group thermochemistry, kinetics and noncovalent interactions†

Lars Goerigk, D^{*a} Andreas Hansen, D^b Christoph Bauer, D^b Stephan Ehrlich, #^b Asim Najibi D^a and Stefan Grimme D^{*b}

https://doi.org/10.1039/C7CP04913G http://www.thch.uni-bonn.de/GMTKN55

GMTKN55 benchmark continued

Final WTMAD-2 values over the entire GMTKN55 for all assessed 83 dispersion-corrected DFAs (kcal mol $^{-1}$). The red bars indicate the three best Fig. 8 approaches on their respective rung of Jacob's Ladder. The suffix "D3" was omitted in all cases, unless it is needed to avoid ambiguity.

GMTKN55 benchmark continued

Fig. 2 The effect of dispersion corrections on WTMAD-1 values (kcal mol⁻¹) for the thermochemistry and kinetics categories of GMTKN55 and for the Fig. 2 entire database.

MOLECULAR PHYSICS, 2017 VOL. 115, NO. 19, 2315-2372 https://doi.org/10.1080/00268976.2017.1333644

TOPICAL REVIEW

a OPEN ACCESS Check for updates

Thirty years of density functional theory in computational chemistry: an overview and extensive assessment of 200 density functionals

Narbe Mardirossian^a and Martin Head-Gordon^{a,b}

https://doi.org/10.1080/00268976.2017.1333644

- · Another **very good benchmark** for main-group thermochemistry
- · **Includes a comprehensive discussion about DFT**, *e.g.*, on functional ingredients
- From abstract:
	- \circ "Ultimately, today's state-of-the-art functionals are close to achieving the level of accuracy desired for a broad range of chemical applications, and **the principal remaining limitations are associated with systems that exhibit significant self-interaction/delocalisation errors and/or strong correlation effects.**"

Figure 17. RMSDs in kcal/mol […] for the 20 density functionals featured in this review, along with the minimum RMSD from across all 200 benchmarked functionals […] NCED stands for non-covalent dimers (easy), NCEC stands for non-covalent clusters (easy), NCD stands for non-covalent dimers (difficult), IE stands for isomerisation energies (easy), ID stands for isomerisation energies (difficult), TCE stands for thermochemistry (easy), TCD stands for thermochemistry (difficult), and BH stands for barrier heights.

CHEMICAL REVIEWS^SCite This: Chem. Rev. 2019, 119, 10856-10915

Review pubs.acs.org/CR

Quantum Chemistry in the Age of Quantum Computing

Yudong Cao, $\stackrel{\dagger,\ddagger}{\downarrow}$ Jonathan Romero, $\stackrel{\dagger,\ddagger}{\downarrow}$ Jonathan P. Olson, $\stackrel{\dagger,\ddagger}{\downarrow}$ Matthias Degroote, $\stackrel{\dagger,\mathbb{I},\mathbb{S}}{\uparrow}$ Peter D. Johnson, $\stackrel{\dagger,\ddagger}{\downarrow}$ Mária Kieferová, $\stackrel{\mathbb{II},\bot,\ddagger}{\downarrow}$ Ian

ABSTRACT: Practical challenges in simulating quantum systems on classical computers have been widely recognized in the quantum physics and quantum chemistry communities over the past century. Although many approximation methods have been introduced, the complexity of quantum mechanics remains hard to appease. The advent of quantum computation brings new pathways to navigate this challenging and complex landscape. By manipulating quantum states of matter and taking advantage of their unique features such as superposition and entanglement, quantum computers promise to efficiently deliver accurate results for many important problems in quantum chemistry, such as the electronic structure of molecules. In the past two

decades, significant advances have been made in developing algorithms and physical hardware for quantum computing, heralding a revolution in simulation of quantum systems. This Review provides an overview of the algorithms and results that are relevant for quantum chemistry. The intended audience is both quantum chemists who seek to learn more about quantum computing and quantum computing researchers who would like to explore applications in quantum chemistry.

Helsinki Winter School in Theoretical Chemistry 2020

This is the 36th Winter School

QC-4C: Quantum Computers for Chemistry

Helsinki, 14-17 December 2020

Organised by the University of Helsinki and CSC

Summary

Computational methods and simulations can be *immensely useful* for explaining, predicting, and designing chemistry and chemical phenomena

One has to know the methods well, however!

• What are the strengths *and limitations* of a specific methodology?

The computer will almost always give you a number, it is up to you as a scientist to assess the reliability of the result!

Further reading

- · The surface was barely scratched; **For more detail, the following text books are excellent**
- · Frank Jensen, "Introduction to Computational Chemistry"
	- o Great overview of QC methods, as well as the basics of MM
- · Wolfram Koch, Max C. Holthausen, "A Chemist's Guide to Density Functional Theory"
	- o Fundamentals of DFT from a chemical viewpoint
- · Kieron Burke *et al,* "The ABC of DFT", *http://dft.uci.edu/research.php*
	- o A more in-depth treatment of DFT. Preliminary version but already good
- · Trygve Helgaker, Poul Jørgensen, Jeppe Olsen, "Molecular Electronic-Structure Theory"
	- o Very detailed account of correlated wave-function methods
- · Steven M. Bachrach, "Computational Organic Chemistry"
	- o Brief intro of methods, followed by examples relevant for organic chemistry

Three recent overviews of different density functionals:

- · Mardirossian & Head-Gordon (2017) "Thirty years of density functional theory in computational chemistry: an overview and extensive assessment of 200 density functionals" *https://doi.org/10.1080/00268976.2017.1333644*
- · Goerigk *et al.* (2017) "A look at the density functional theory zoo with the advanced GMTKN55 database for general main group thermochemistry, kinetics and noncovalent interactions" *https://doi.org/10.1039/C7CP04913G*
- · Dohm *et al.* (2018) "Comprehensive Thermochemical Benchmark Set of Realistic Closed-Shell Metal Organic Reactions" *https://doi.org/ 10.1021/acs.jctc.7b01183*

Good luck!